

# Theory of current-driven motion of Skyrmions and spirals in helical magnets

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We study theoretically the dynamics of the spin textures, i.e., Skyrmion crystal (SkX) and spiral structure (SS), in two-dimensional helical magnets under external current. By numerically solving the Landau-Lifshitz-Gilbert equation, it is found that (i) the critical current density of the motion is much lower for SkX compared with SS in agreement with the recent experiment, (ii) there is no intrinsic pinning effect for SkX and the deformation of the internal structure of Skyrmion reduces the pinning effect dramatically, (iii) the Bragg intensity of SkX shows strong time-dependence as can be observed by neutron scattering experiment.

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It has been recognized for a long time that the spin polarized electric current drives the motion of spin textures due to the spin transfer torque since the original theoretical proposal by ref.[1, 2]. The experimental demonstrations of this effect in the spin valves and ferromagnetic domain wall systems followed [3], which stimulated recent active researches on this phenomenon including those toward the applications such as racetrack memory [4]. However, the threshold current density  $j_c$  for the domain wall motion is rather high of the order of  $10^{10} - 10^{12}$  A/m<sup>2</sup> [3]. Therefore, the Joule heating is a serious issue, and usually the experiments have been done by using the short pulses of electric current. There have been two origins of the pinning effect on the domain wall motion. One is the intrinsic pinning due to the magnetic anisotropy [5]. More detailed analysis has showed that this intrinsic pinning occurs when only the Gilbert damping  $\alpha$  is taken into account, and in the generic situation of the finite  $\beta$  term there is no intrinsic pinning while the velocity is reduced [6]. When  $\alpha = \beta$ , there is completely no intrinsic pinning [7]. Another origin is the extrinsic pinning due to the imperfections such as impurities and defects. In realistic situation, these two mechanisms are entangled and it is of vital importance to find some magnetic systems with lower critical current density.

From this viewpoint, it is important to look for other types of magnetic textures for the current driven motion. The Skyrmion [8], a topological spin texture where the spins point in all directions wrapping a sphere, is an interesting candidate in this respect. In magnets with non-centrosymmetric crystal structures, the Dzyaloshinskii-Moriya (DM) interaction [9, 10] is allowed which naturally leads to the spiral spin structure. Under an external magnetic field  $\vec{B}$ , this spiral spins turns into the trian-

gular crystal of Skyrmions in the narrow region in the  $B - T$  phase diagram near the transition temperature  $T_c$  in 3D as observed in neutron scattering experiments [11], and over a wide region of  $B - T$  diagram in 2D as theoretically predicted [12] and observed experimentally by Lorentz microscope [13]. Remarkably, the small density of charge current ( $j_c \sim 10^6$  A/m<sup>2</sup>) is found to drive the motion of Skyrmions in 3D MnSi crystal as indicated by the slight rotation of the Bragg spot of the Skyrmion crystal in neutron scattering experiment [14]. Furthermore, the current driven Skyrmion motion with even smaller  $j_c \sim 10^5$  A/m<sup>2</sup> is concluded by the real space observation by Lorentz microscope in FeGe thin film [15]. These observations have opened a new route to the manipulation of magnetic structures by ultra-low current density, and it is an important issue to study the mechanism of the pinning and the current-induced Skyrmions dynamics, which we undertake in the present letter.

The solid angle subtended by the spins in Skyrmion structure produces a fictitious magnetic field acting on the conduction electrons [16], which gives rise to the topological Hall effect [17, 18]. The motion of the Skyrmion produces the motion of this effective magnetic field, and hence the electromagnetic induction leading to an effective electric field in the transverse direction to the current. This results in a change in the Hall voltage when the Skyrmion motion sets in as predicted theoretically [16] and observed experimentally [19]. This topological nature of Skyrmions also affects their dynamics since the coordinates,  $X$  and  $Y$ , of the skyrmion core are canonical conjugate. Actually in ref.[16], the collective phonon modes and the pinning of the Skyrmion crystal have been studied, and a small threshold current density has been concluded. However, since the internal distortion of the Skyrmions has not been considered, it failed to explain the drastic difference between the pinning effect in spiral phase and that in Skyrmion phase, which was clearly shown in the experiment [15].

We start with the following classical Heisenberg Hamiltonian in a two-dimensional square lattice, which includes

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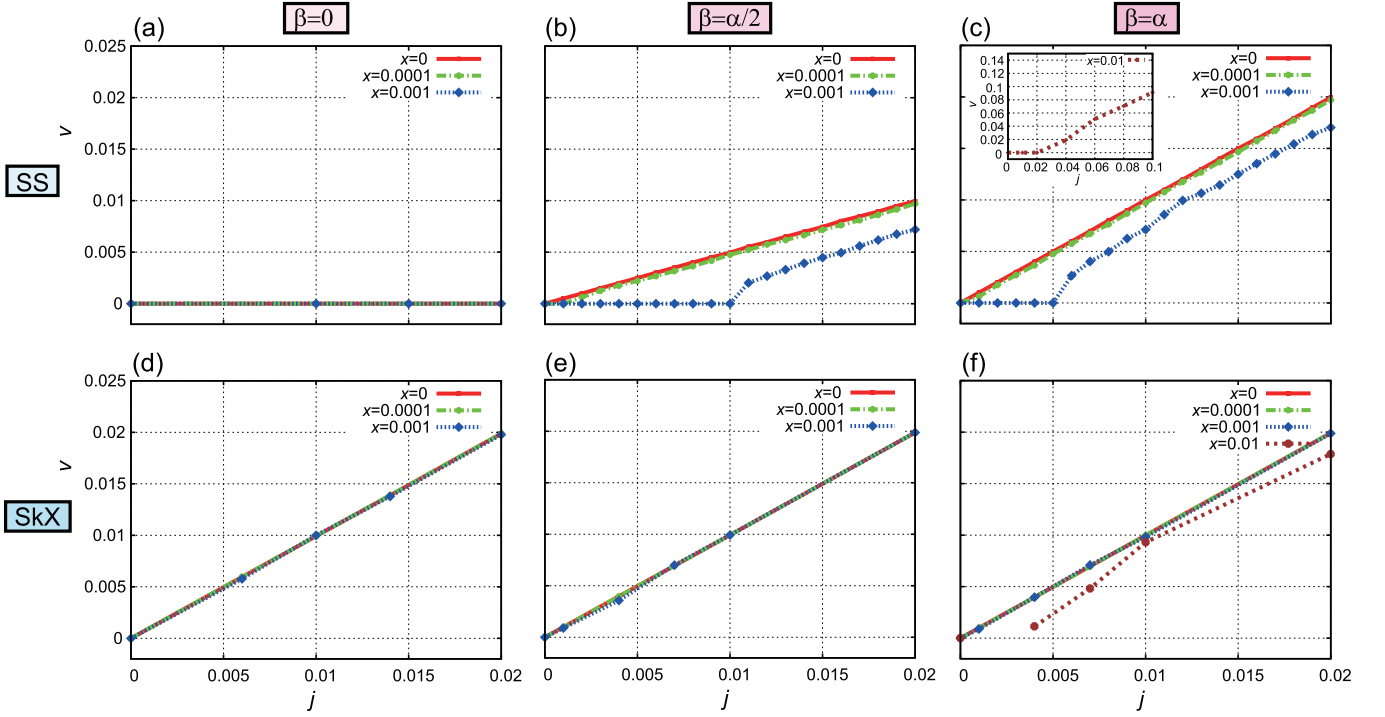


FIG. 1: The relation between the current density  $j$  and velocity  $v$ . Impurity density  $x$  is  $x = 0, 0.0001, 0.001$  for  $\beta = 0, \alpha/2$ , and  $x = 0, 0.0001, 0.001$  for  $\beta = \alpha$ . (a)  $\beta = 0$  in SS. (b)  $\beta = \alpha/2$  in SS. (c)  $\beta = \alpha$  in SS. Note that the axis-ranges of the inset is different from that of outer one. (d)  $\beta = 0$  in SkX. (e)  $\beta = \alpha/2$  in SkX. (f)  $\beta = \alpha$  in SkX.

the ferromagnetic exchange interaction, DM interaction, Zeeman coupling with an external magnetic field, and magnetic anisotropy with an easy axis in a direction perpendicular to a lattice-plane due to the randomly-distributed imperfections:

$$\begin{aligned} \mathcal{H} = & -J \sum_{\vec{r}} \vec{M}_{\vec{r}} \cdot \left( \vec{M}_{\vec{r}+\vec{e}_x} + \vec{M}_{\vec{r}+\vec{e}_y} \right) \\ & - D \sum_{\vec{r}} \left( \vec{M}_{\vec{r}} \times \vec{M}_{\vec{r}+\vec{e}_x} \cdot \vec{e}_x + \vec{M}_{\vec{r}} \times \vec{M}_{\vec{r}+\vec{e}_y} \cdot \vec{e}_y \right) \\ & - \vec{B} \cdot \sum_{\vec{r}} \vec{M}_{\vec{r}} - A \sum_{\vec{r} \in I} M_z^2. \end{aligned} \quad (1)$$

Here  $I$  denotes the positions of impurities. The parameters are set as  $J = 1$ ,  $D = 0.18$ , and  $A = 0.2$ . The external magnetic field  $\vec{B}$  is put as  $\vec{B} = 0.015 \vec{e}_z$  for the Skyrmion crystal, and  $\vec{B} = 0$  for the helical phase. We then study the spin dynamics at  $T = 0$  by numerically solving the Landau-Lifshitz-Gilbert equation including the terms which represent the coupling between spins and spin-polarized electric current density  $\vec{j}$ :

$$\begin{aligned} \frac{d\vec{M}_{\vec{r}}}{dt} = & \gamma \vec{M}_{\vec{r}} \times B_{\vec{r}}^{\text{eff}} - \frac{\alpha}{M} \vec{M}_{\vec{r}} \times \frac{d\vec{M}_{\vec{r}}}{dt} - \frac{pa^3}{2eM} \left( \vec{j} \cdot \vec{\nabla} \right) \vec{M}_{\vec{r}} \\ & - \frac{pa^3\beta}{2eM^2} \left[ \vec{M}_{\vec{r}} \times \left( \vec{j} \cdot \vec{\nabla} \right) \vec{M}_{\vec{r}} \right], \end{aligned} \quad (2)$$

where  $B_{\vec{r}}^{\text{eff}} = -\partial\mathcal{H}/\partial\vec{M}_{\vec{r}}$ ,  $\gamma$  is the gyromagnetic ratio,  $p$  is the spin-polarizability, and  $a$  is the lattice constant.

The Gilbert damping factor  $\alpha$  is fixed to be  $\alpha = 0.04$ , and three values for  $\beta$  are considered, i.e.,  $\beta = 0, \alpha/2, \alpha$ . We used the fourth-order Runge-Kutta method to solve this differential equation. The size of the sample is  $288 \times 288$  and the periodic boundary condition is used. The initial spin configuration is obtained by the Monte-Carlo method and by further relaxing them at  $j = 0$  in the LLG simulation. After the sufficient convergence of the spin configuration, we switch on a steady electric current and observe the time evolution of the spins. For SS the direction of electric current is put to be parallel to the wave vector of the spiral. In the following context, unit of time is  $\tau \equiv \hbar/J$ , and unit of current density is  $\kappa \equiv \frac{2eSJ}{pa^2\hbar}$ .  $\tau$  and  $\kappa$  are  $\tau \simeq 6.5 \times 10^{-13}$  and  $\kappa \simeq 2.0 \times 10^{13}$  for the typical set of parameters  $J = 1$  meV,  $a = 50$  nm and  $p = 0.1$ .

Figure 1 shows the the mean velocity  $v$  as a function of the electric current density  $j$  for SS and SkX with different values of  $\beta$ . With  $\beta = 0$ , the SS cannot move even without the impurity or defect within the range of the current density of the present study as shown in Fig. 1(a). In sharp contrast, SkX moves freely with the velocity proportional to the current density for all values of the impurity concentration  $x$  and  $\beta$ , even though a slight decrease of  $v$  is observed for  $x = 0.01$  (Figs. 2(d)-(f)). These results clearly indicate that the very strong intrinsic pinning exists for SS while no intrinsic pinning exists for SkX. The motion of the domain wall or SS is asso-

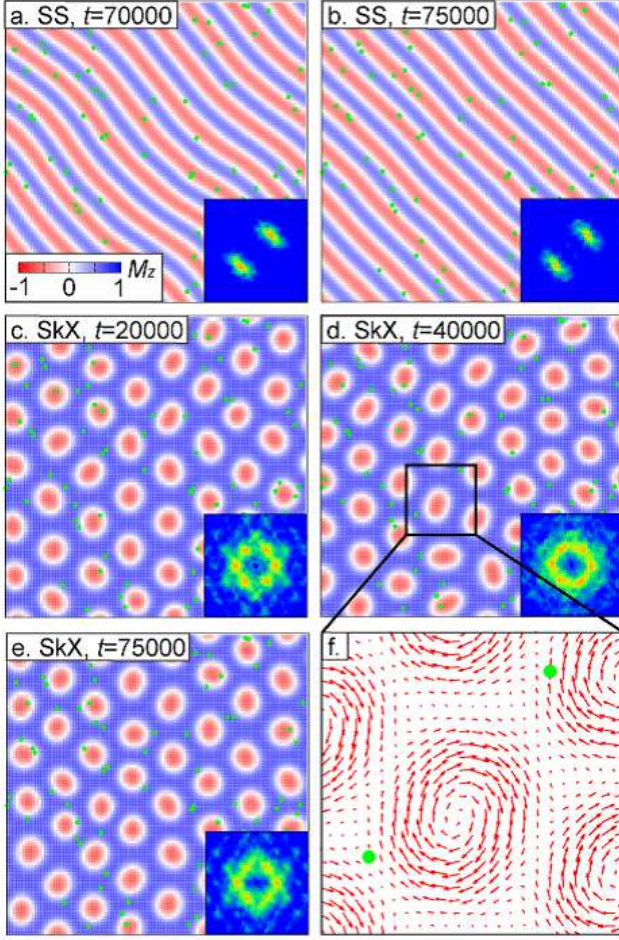


FIG. 2: Snapshots of the moving spin textures at different times. Current density  $j$  and impurity density  $x$  are  $j = 0.006, x = 0.001$  for SS, and  $j = 0.004, x = 0.001$  for SkX. (a) SS,  $t=70000$ . (b) SS,  $t=75000$ . (c) SkX,  $t=20000$ . (d) SkX,  $t=40000$ . (e) SkX,  $t=75000$ . (f) The partial enlarged picture of (e).

ciated with the rotation of the spins within the plane, which is generated by the spin component perpendicular to them. Therefore, one needs the finite current density to compete with the magnetic anisotropy energy for the tilting of the spins away from the spin rotation plane [5]. Usual intrinsic pinning comes from anisotropy with an easy plane. The system now considered also has an easy plane anisotropy because of DM interaction. The easy plane is perpendicular to the wave vector of the SS as determined by the vector  $\vec{D}$  of the DM interaction. On the other hand, there is no intrinsic pinning for SkX because of the different canonical conjugate relation for the collective coordinates. Namely, the X and Y components of the center of mass motion of a Skyrmion determines the equation of motion [16], and hence uniform motion of the SkX does not require the distortion of the spin structure.

For finite  $\beta$ , the intrinsic pinning does not give the finite threshold current density and especially in the case of  $\beta = \alpha$ , no intrinsic pinning is expected [7], where the pinning effect is purely the extrinsic one. In these cases, we can see that SS has a threshold current density  $j_c$  which is a decrease function of  $\beta$ . This indicates that the interplay between the intrinsic and extrinsic mechanisms of the pinning occurs for SS. On the other hand, the velocity  $v$  is hardly suppressed by the impurities for  $x = 0.0001, 0.001$  from the case of no impurity. Therefore, we can conclude that there is a large difference in the mechanism of the extrinsic pinning among these two phases, as discussed in depth below. From  $x = 0.01$  in Figs. 1(c) and (f), we can see that the threshold current density in SkX is at least factor 5 smaller than that in SS in the dense impurity case while the ratio is much more in the dilute cases.

In order to study the change in the spin configurations during the motion, we take their snapshots in real space as shown in Fig. 2. Figures 2(a), (b) show the moving SS, where the position of each impurity is indicated by green dots. It is seen that even with  $x = 0.001$ , the characteristic length scale of the distortion is longer than the mean distance between impurities, and hence the collective pinning mechanism applies here [20]. Note that due to the one dimensional nature of the spiral the impurities are always acting on the SS and distort it. This is not the case for SkX as shown in Figs. 2(c)-(f). We can see that each Skyrmion avoids the impurities by deforming the triangular lattice and also transforming its shape. This is because the region between Skyrmions are ferromagnetic state which does not feel the pinning, i.e., there is no energy change with the translation of the ferromagnetic spin configuration. This kind of motion is a key feature of SkX for explaining their ultra-low threshold current density. When the impurity works as an attractive center, the Skyrmion tends to rotate around it rather than approaching to it. All of these features, i.e., the distortion of the triangular lattice and each Skyrmion, and the rotating motion around the impurities, work together to reduce the pinning effect for SkX.

Deformation of SkX explained above should be able to be detected experimentally. For that purpose, we propose the neutron scattering experiment on the time-dependence of the Bragg peaks of the SkX. Figure 3 shows the change in the peak value of the spin structure function  $S(\vec{k})$  at the Bragg points for the SkX. The values are normalized so that the integral of  $S(\vec{k})$  over the first Brillouine zone is 1. Because of the uniform magnetization under the external magnetic field, the component at  $\vec{k} = (0,0)$  is finite, and hence the red line for  $x = 0$  corresponds to the perfect SkX. For finite  $x$ , the peak value fluctuates in time. This means that there is the time-dependent deformation of SkX. When we take the Fourier transformation (Fig. 3), there are two characteristic frequencies. One is around  $\omega = 0.045\pi (= 0.1413)$ , which corresponds to the periodic distortion and recovery of SkX synchronized with its motion with one lattice con-

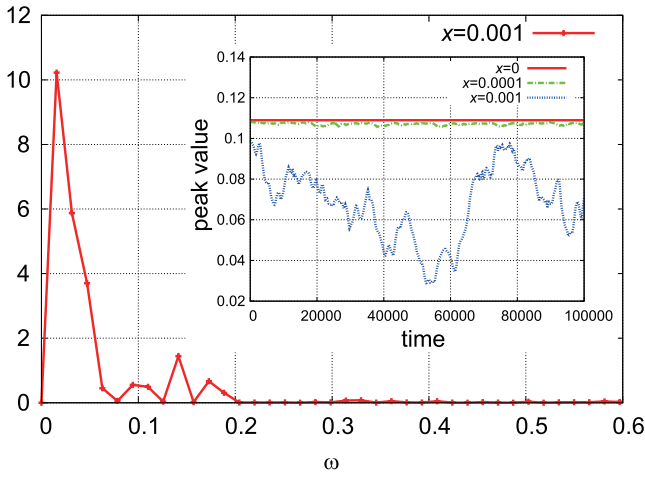


FIG. 3: The inset of the figure is the time-dependence of the Bragg peak value in SkX ( $j = 0.004$ ). The outer figure is the spectra of  $x = 0.001$ .

stant of SkX. It is because when a Skyrmion approaches imperfections the triangular lattice is little broken, but soon after the Skyrmion passed by imperfections the lattice begins to return to triangular lattice. The other lower one  $\omega < 0.005\pi (= 0.0157)$  corresponds to the slow and collective distortion of SkX.

In summary, we have theoretically studied the dynamics of spiral spin structure (SS) and Skyrmion crystal

(SkX) in helical magnets with impurities or defects driven by electric currents. We have confirmed the strong intrinsic pinning effect for SS, while no intrinsic pinning for SkX. Also the extrinsic pinning effect is found to be much weaker for SkX compared with SS. By the real-space snapshots, the origin of this difference is revealed; SkX avoid the imperfections by the deformation of the triangular lattice and also the shape of each Skyrmion together with the rotating motion around the impurity. This leads to the very weak threshold current density for SkX as observed experimentally compared with SS. Finally, we have proposed the neutron scattering experiment to detect the time-dependent Bragg peaks showing the periodic change in the SkX.

During the completion of this paper we became aware of a recent relevant paper by Liu and Li [21], which addresses the pinning mechanism of Skyrmions in chiral magnets. They consider the one and two Skyrmions with one pinning center, while the present work focuses on the crystal of Skyrmions with many pinning centers. We are grateful for the insightful discussions with Prof. Y. Tokura and Dr. X.Z. Yu. This work is supported by Grant-in-Aids for Scientific Research (No. 24224009) from the Ministry of Education, Culture, Sports, Science and Technology of Japan, and also by Funding Program for World-Leading Innovative R&D on Science and Technology (FIRST Program). MM was supported by G-COE Program “Physical Sciences Frontier” from MEXT Japan.

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